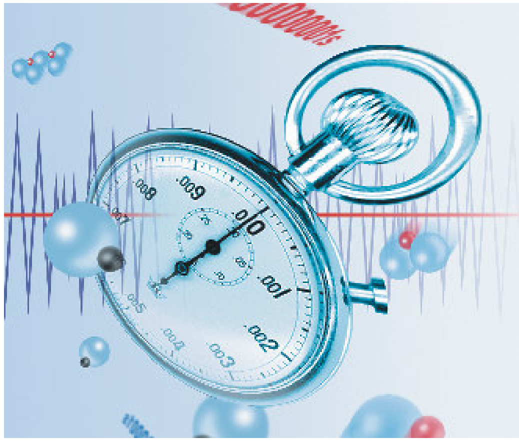
Modern frequency counting principles



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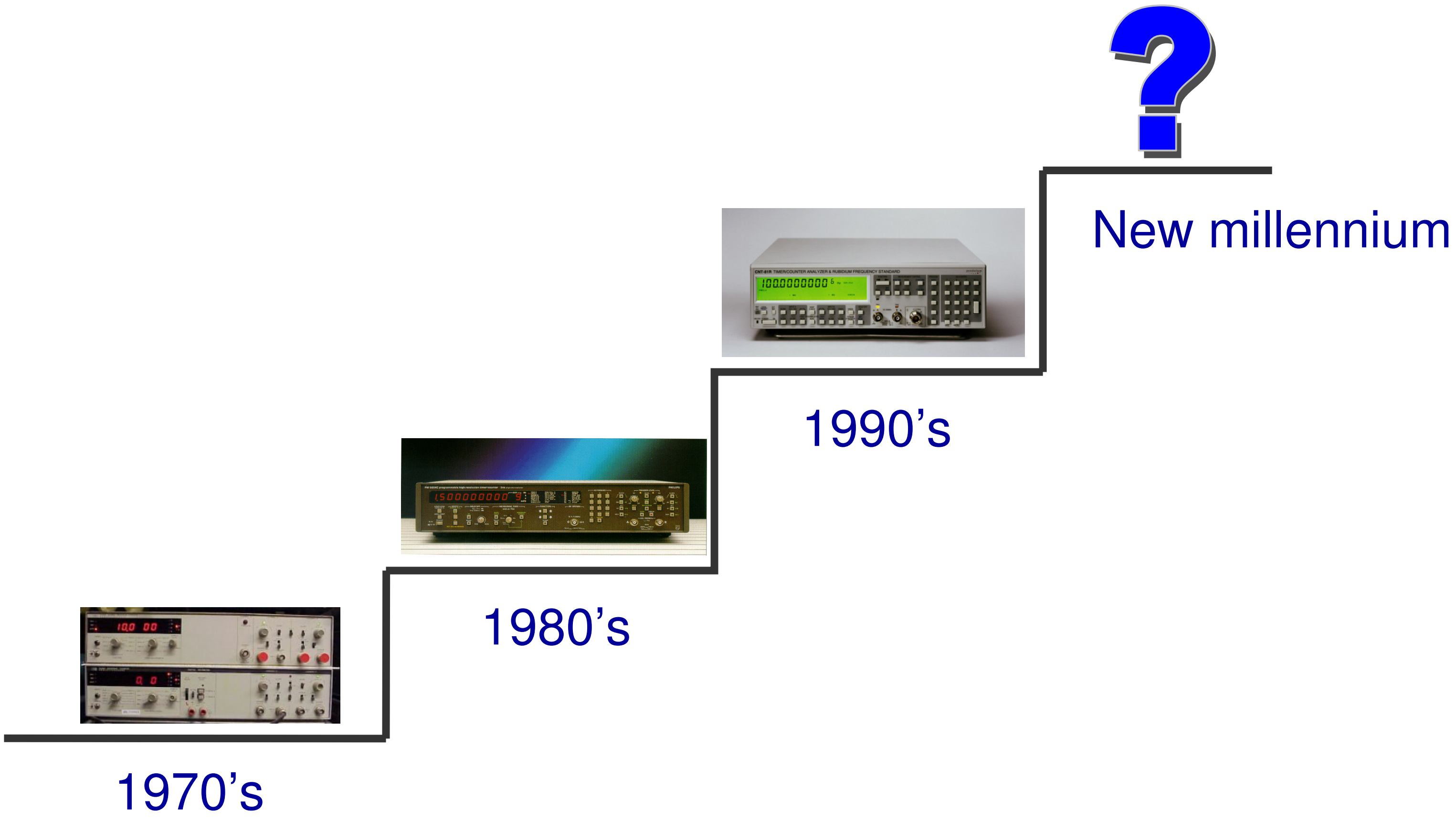
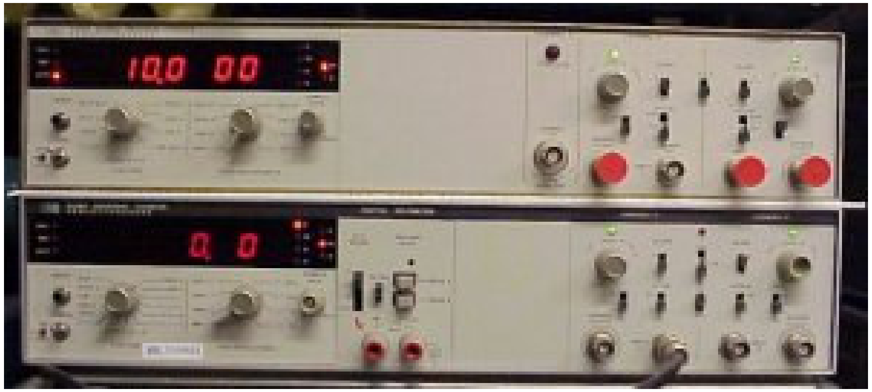
Frequency counter evolution



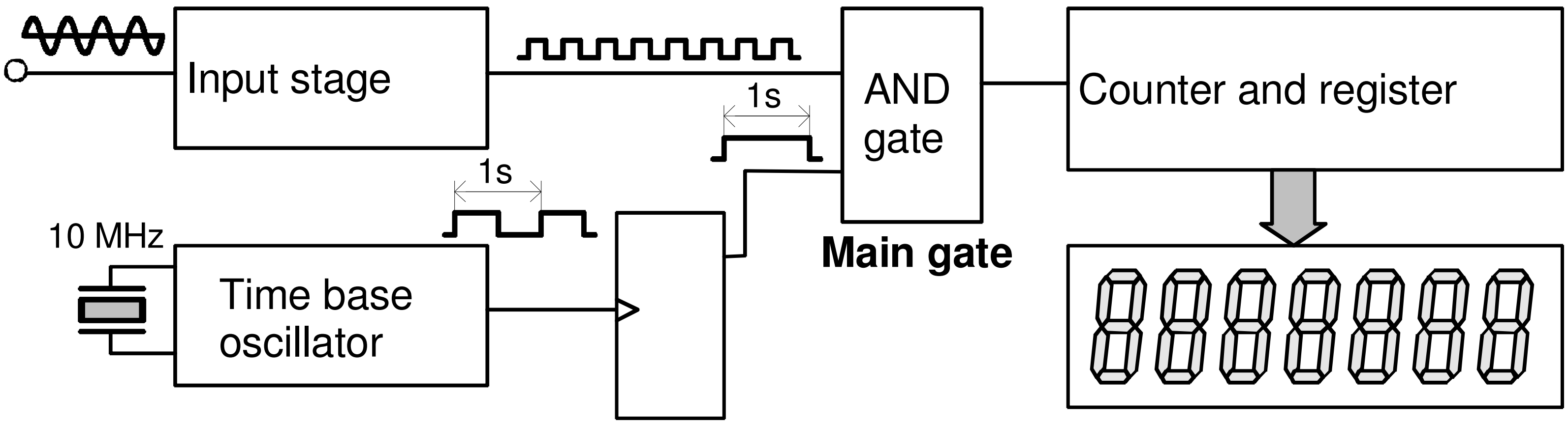
Evolution - 1970's



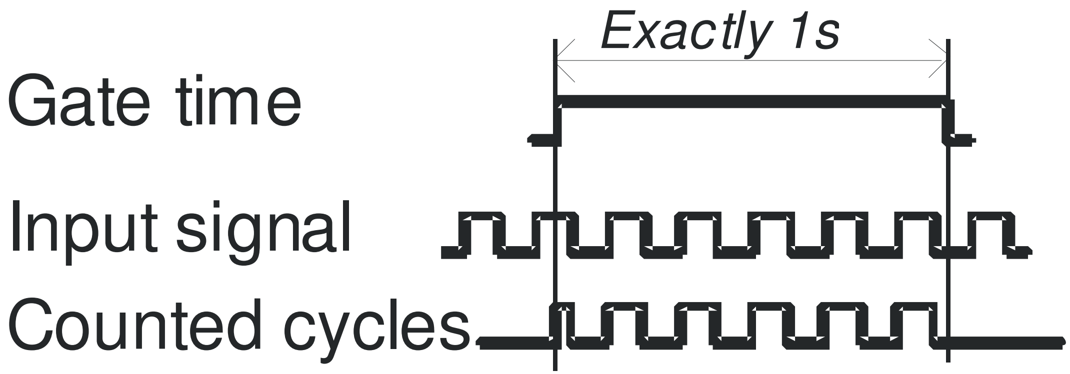
HP 5328A

- Conventional Counters
- Frequency resolution 2-8 digits/s

Conventional counting

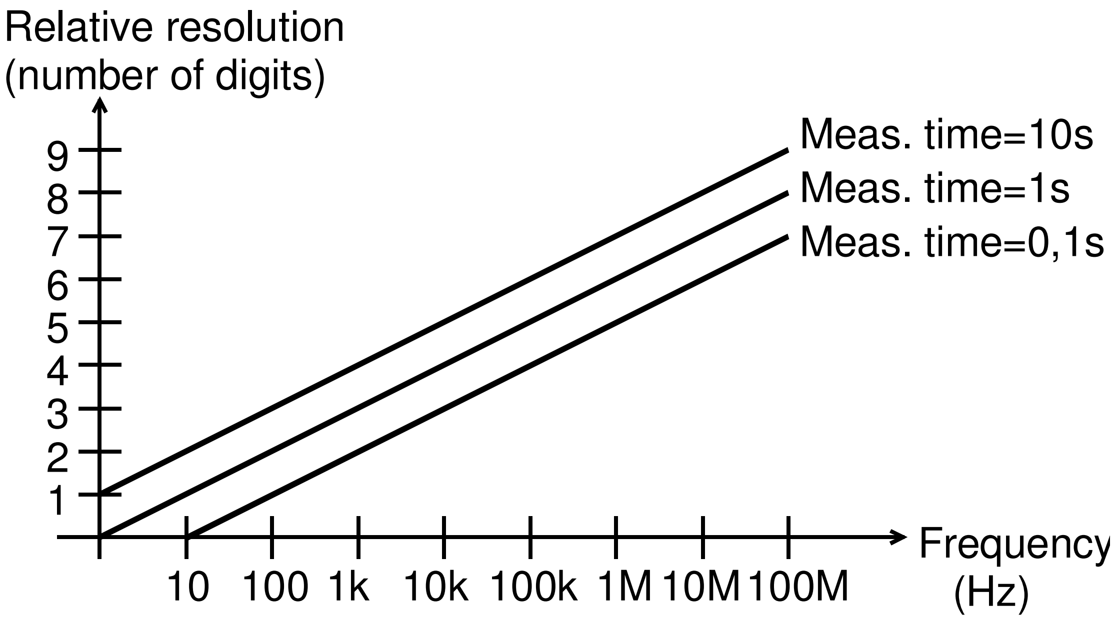


Conventional counting



NOT an integer number of cycles!

Conventional counting-resolution

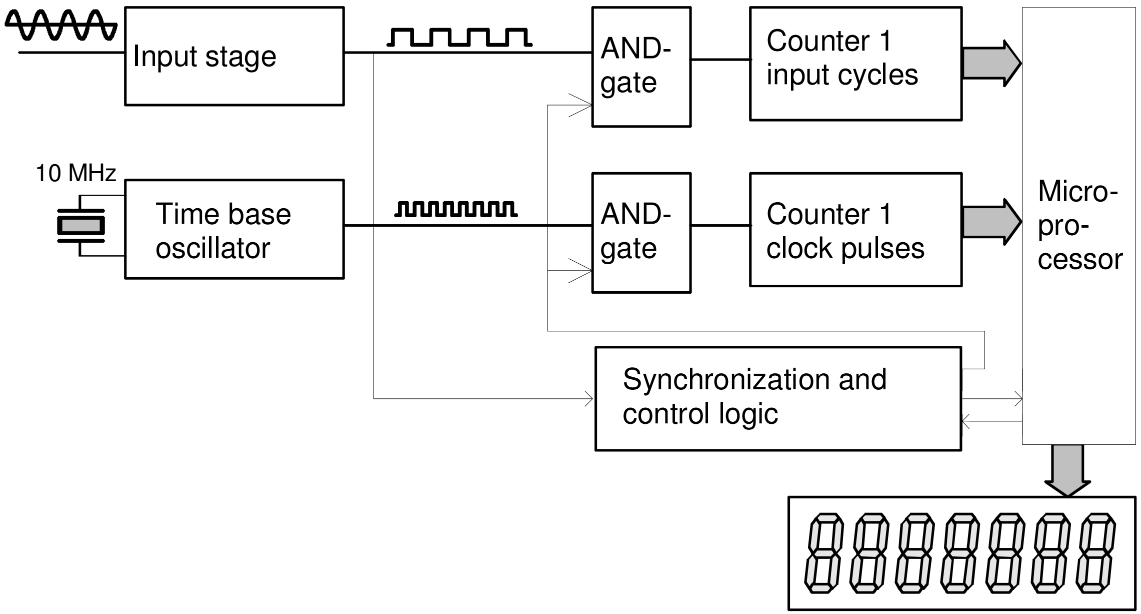


Evolution - 1980’s

PM 6645C

- Reciprocal counters
- Improved frequency resolution 7-9 digits/s

Reciprocal counter - or period meter



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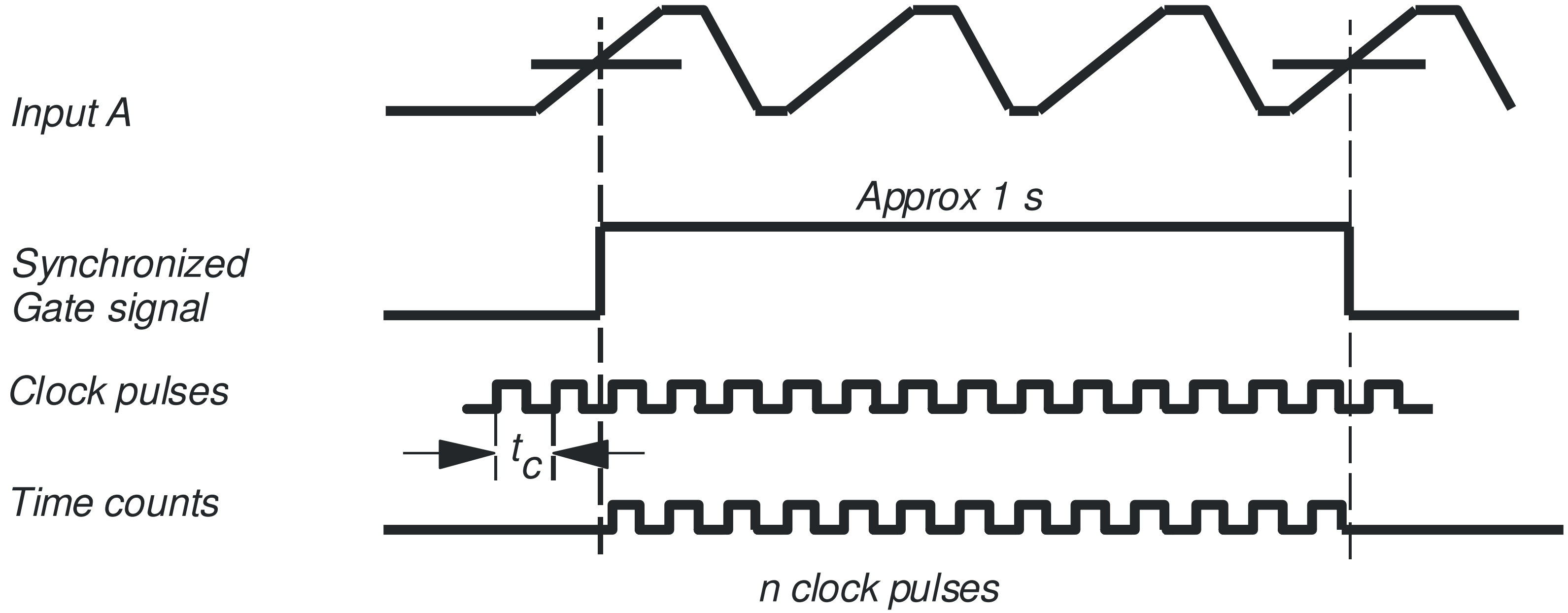
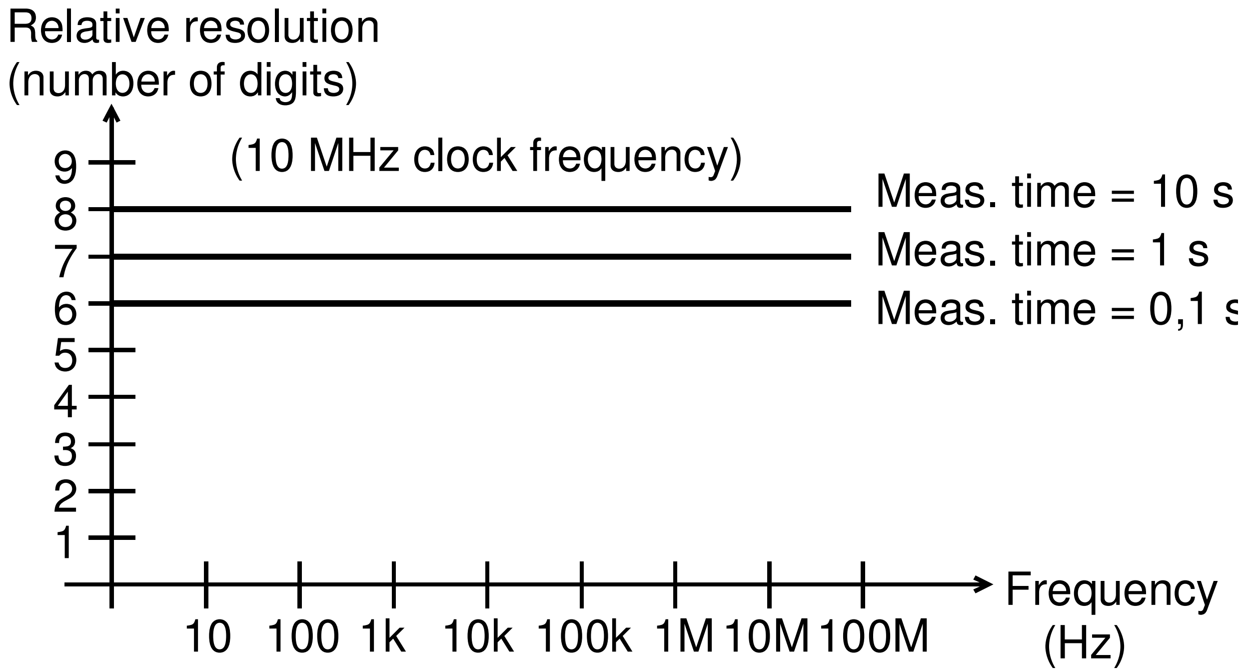
Reciprocal counting



An integer number of input cycles!

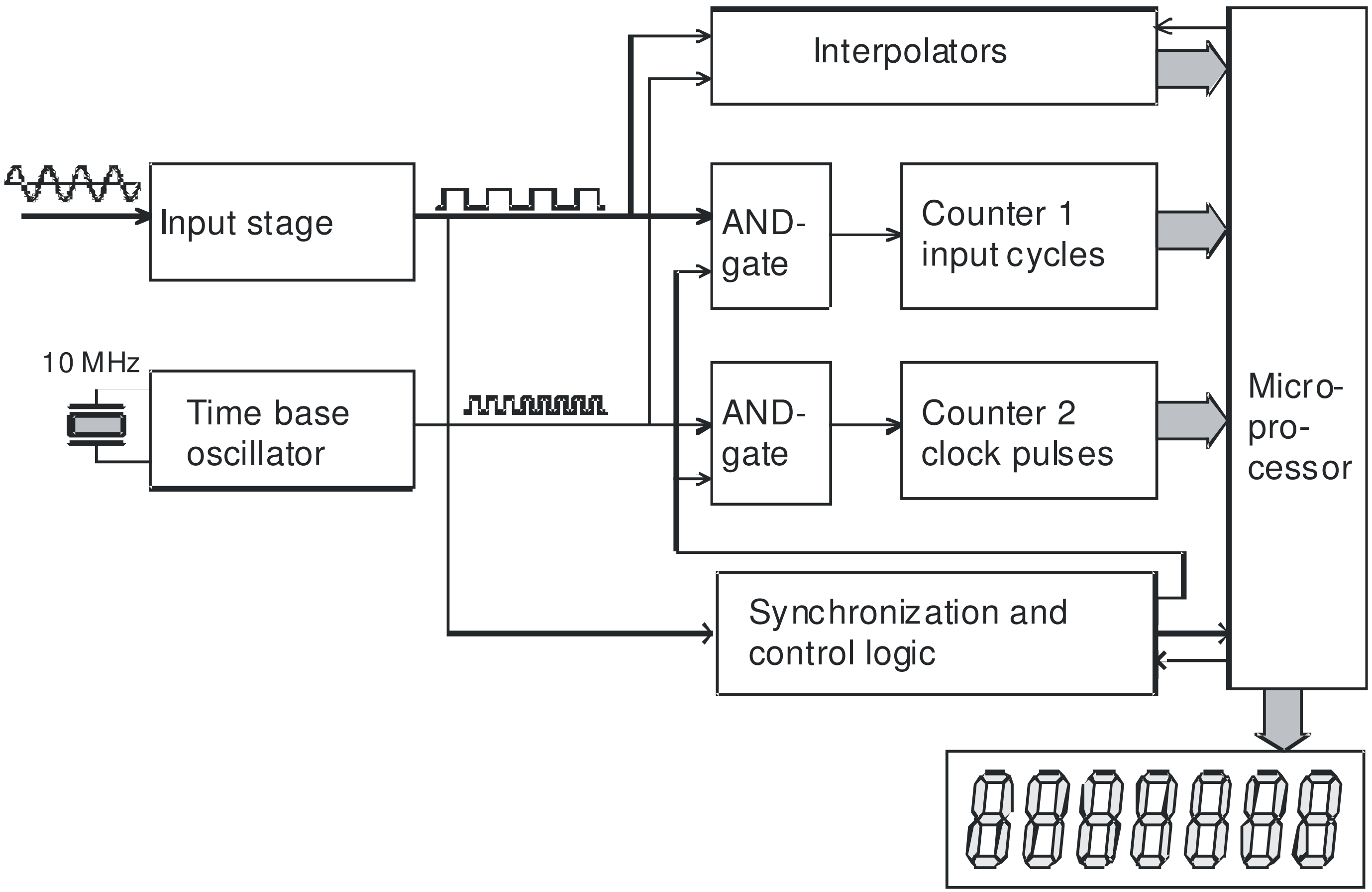
Reciprocal counting - resolution



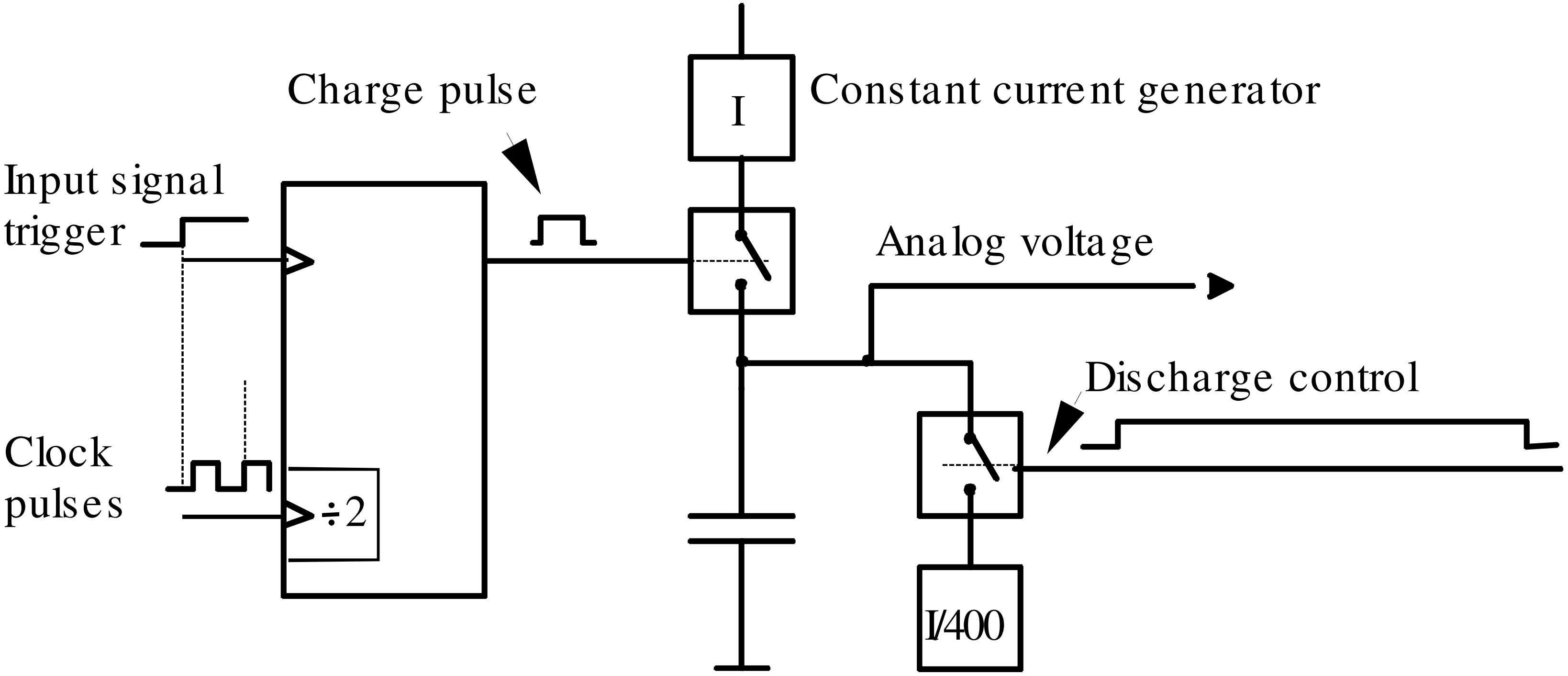
Evolution - 1990's

- Interpolating reciprocal counters
- 9 -11 digits/s frequency resolution

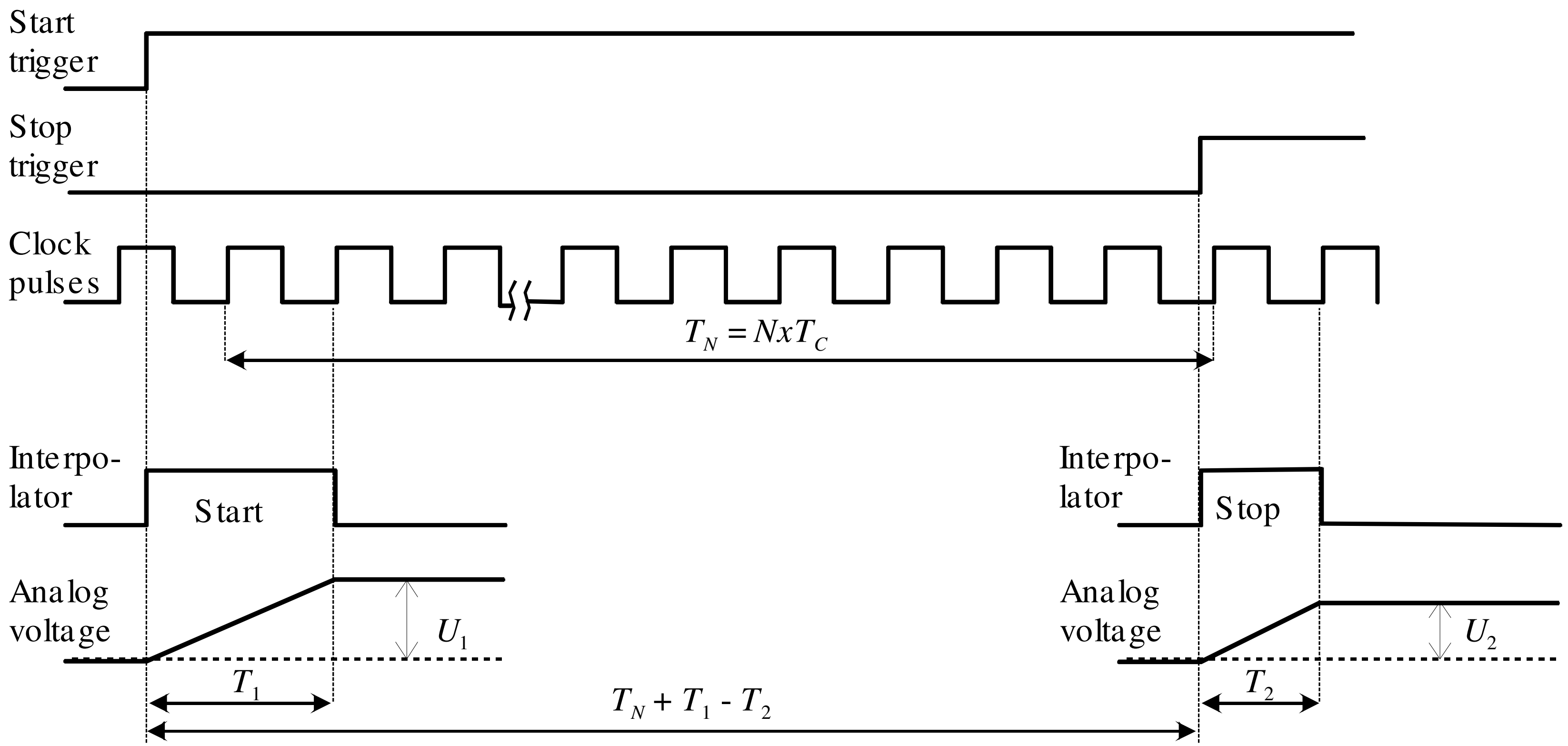
Interpolating counting



Interpolator circuit



Interpolator timing diagram



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Evolution - 2000's

- Continuously timestamping counters
- 10-12 digits/s frequency resolution

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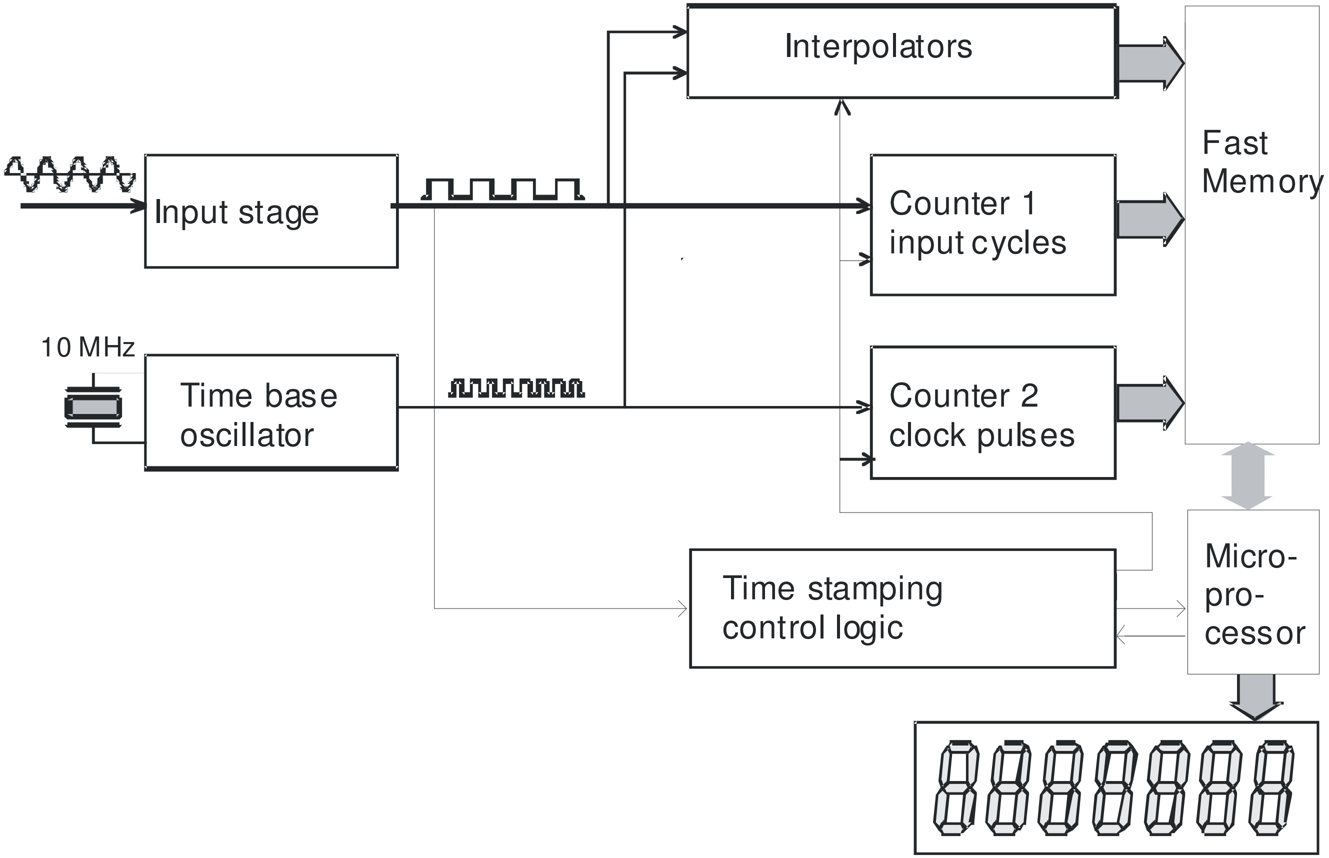
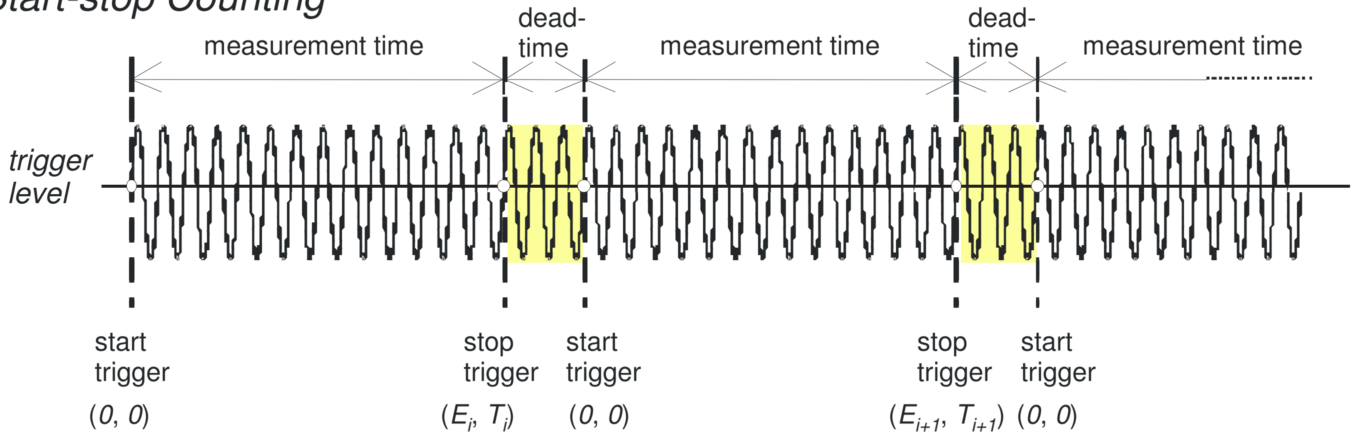
Continuous Timestamping

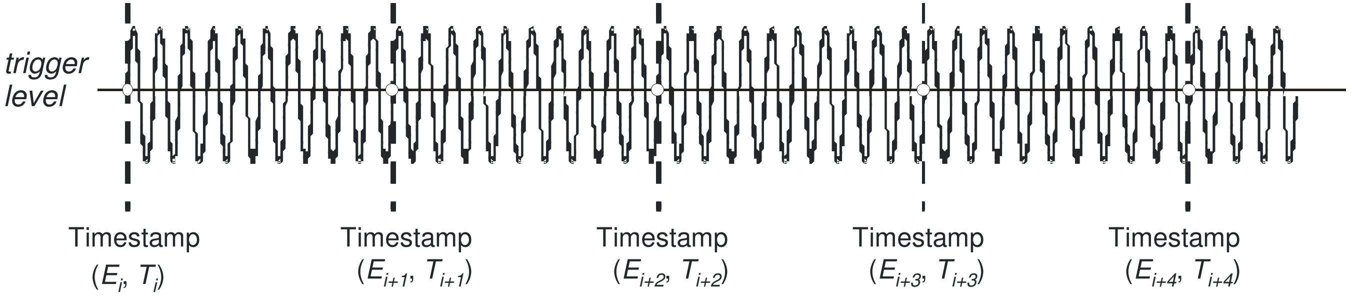


Continuous Timestamping

Start-stop Counting



Continuous Time-stamping Zero dead-time



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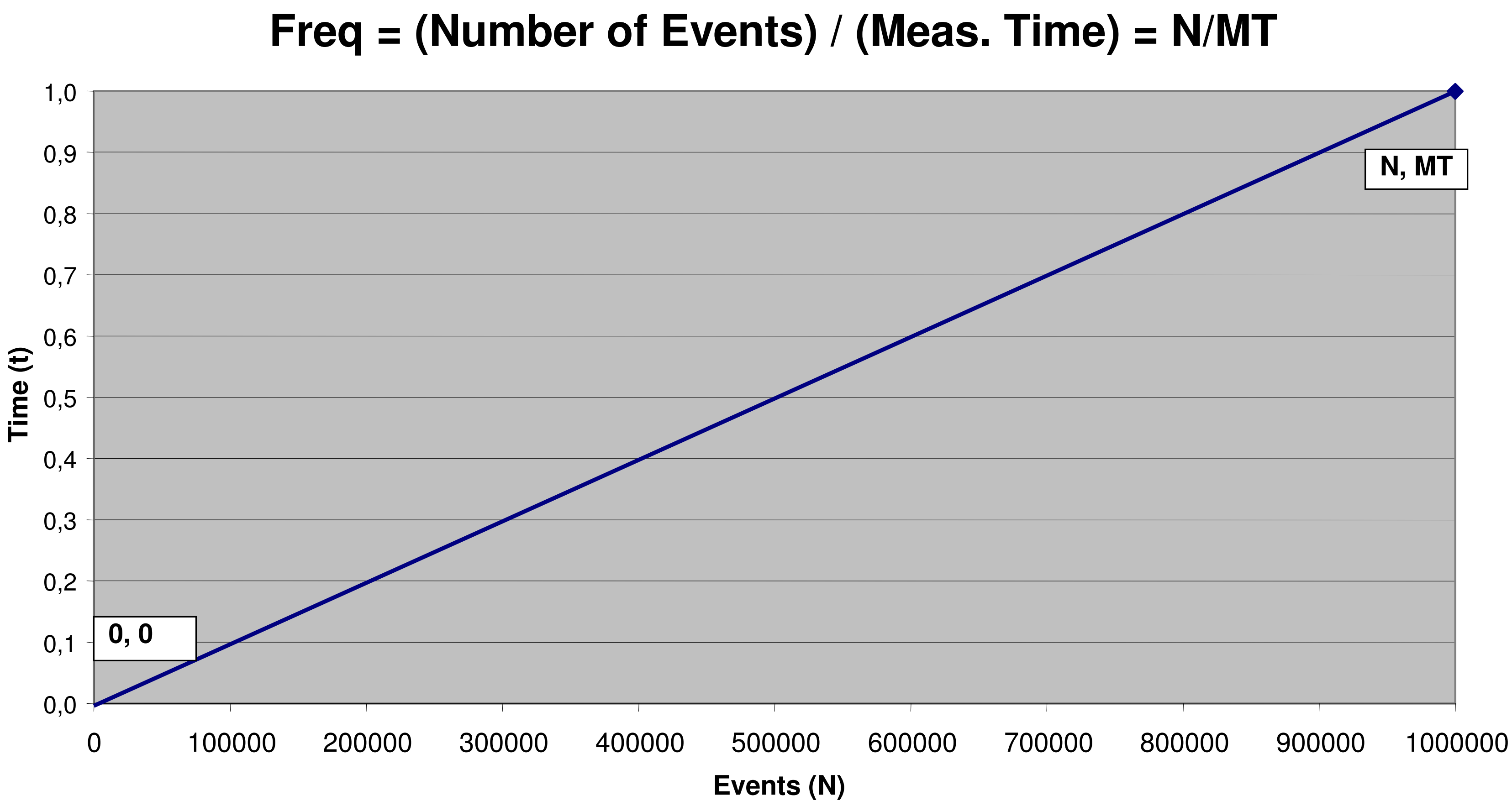
Start-stop counters



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Uncertainty start-stop counting

Start time ($t_1 = 0$) uncertainty: t_{res}

Stop time ($t_2 = MT$) uncertainty: t_{res}

Event count uncertainty (N): zero

Relative frequency uncertainty =

Relative period uncertainty =

$$\frac{\sqrt{(Start\ time\ uncert.)^2 + (Stop\ time\ uncert.)^2}}{Meas\ time} = \frac{t_{RES} \cdot \sqrt{2}}{MT}$$

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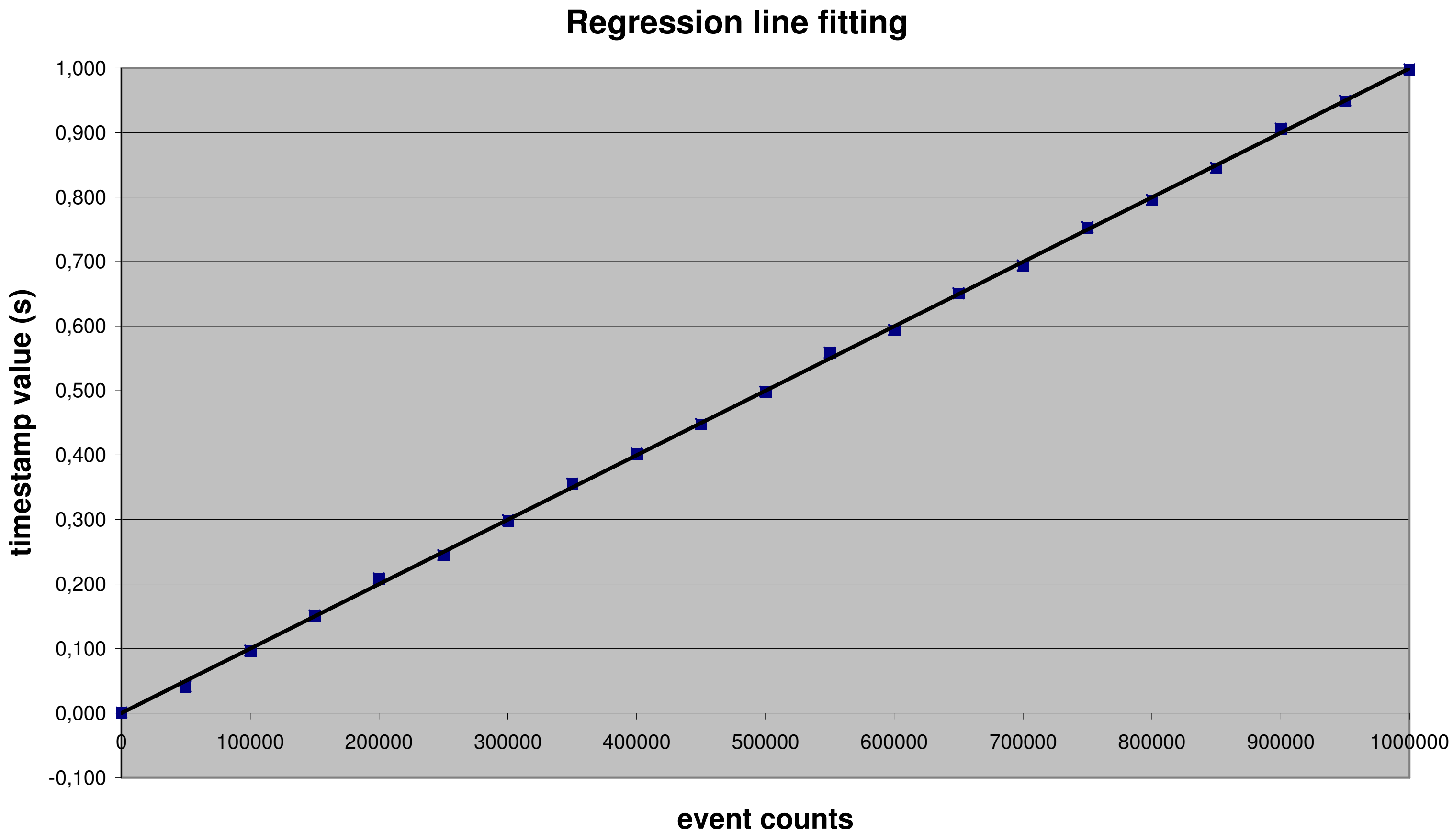
Time stamping counters



Uncertainty timestamping counters

Period = Regression line ($t = a + bE$) slope b

$$b = \frac{n\sum E_k t_k - \sum E_k \sum t_k}{n\sum E_k^2 - \left(\sum E_k\right)^2}$$

Each time stamp (t_i) uncertainty: t_{res}

Each event count (E_i) uncertainty: zero

Period uncertainty = $s^2(b) = \dfrac{s^2(t)}{s^2(E)\cdot(n-2)}$

$s(t) = t_{res}$, but what is $s(E)$???

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Uncertainty timestamping counters

What is $s(E)$???

For n>>1 approximate a rectangular distribution for {E}:

$(E$ distribution between E_0 and $E_0 + N, \quad E_i = E_0 + \frac{i \cdot N}{n})$

$$s(E) \approx \sigma = \frac{N}{2\sqrt{3}}$$

Relative period uncertainty:

$$\frac{s(b)}{b} = \frac{s(T)}{T} = \frac{s(t)}{T \cdot s(E) \cdot \sqrt{n}} = \frac{t_{res}}{\frac{MT}{N} \cdot \frac{N}{2 \cdot \sqrt{3}} \cdot \sqrt{n}} = \frac{2 \cdot \sqrt{3} \cdot t_{res}}{MT \cdot \sqrt{n}}$$

(Period $T = MT/N$)

Statistical improvement of Resolution

- Traditional start-stop (measuring time = MT)
  - Resolution is $\frac{\sqrt{2} \cdot t_{RES}}{MT}$
- Linear regression (n>>1)
  - Resolution is $\frac{2\sqrt{3} \cdot t_{RES}}{MT \cdot \sqrt{n}}$
    - t_{RES} is rms-uncertainty of each time stamp value
- Improvement: $\frac{\sqrt{6}}{\sqrt{n}} \approx \frac{2,45}{\sqrt{n}}$

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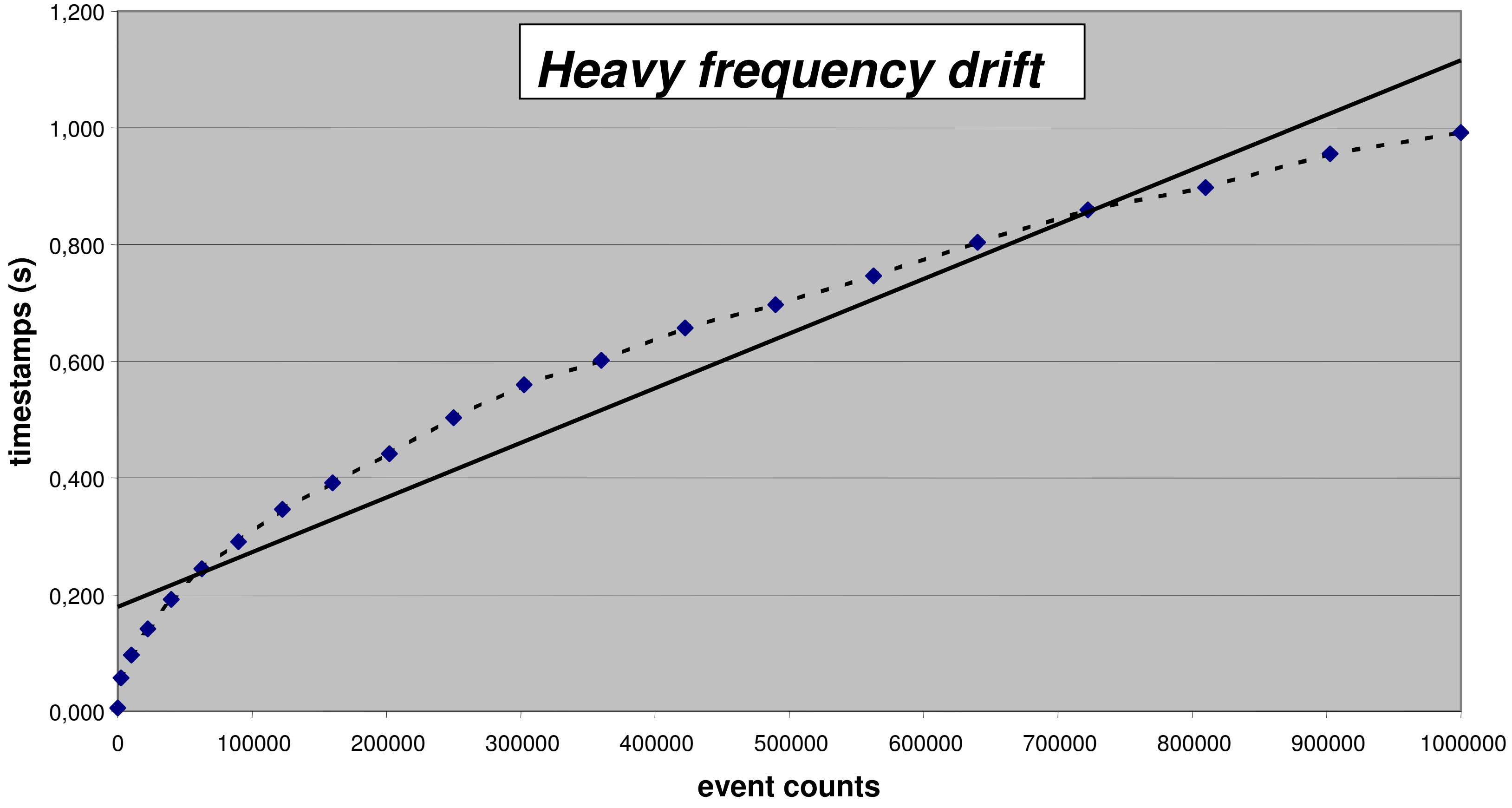
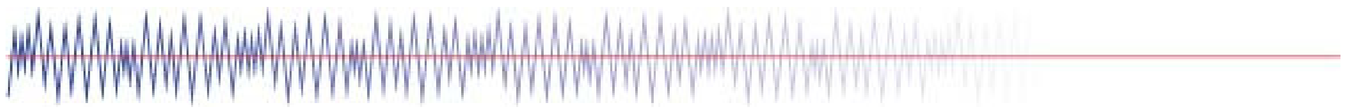
Use regression line method for stable frequencies only!





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Timestamping advantages in CNT-90

- Increased frequency resolution (regression analysis)
- True back-to-back frequency (correct Allan deviation)

Allan Deviation

N frequency samples f_k ($f_1 \ldots f_N$) over τ seconds each, starting at time t_{k-1}. Zero dead-time means: $t_{k+1} = t_k + \tau \; or \; t_k = k \cdot \tau + t_0$

$$Allan\ dev: \sigma_y(\tau) = \sqrt{\frac{1}{2}\left\langle \left(y(t_k + \tau) - y(t_k)\right)^2 \right\rangle}$$

$$(y_k = \frac{f_k - f_{ref}}{f_{ref}})$$

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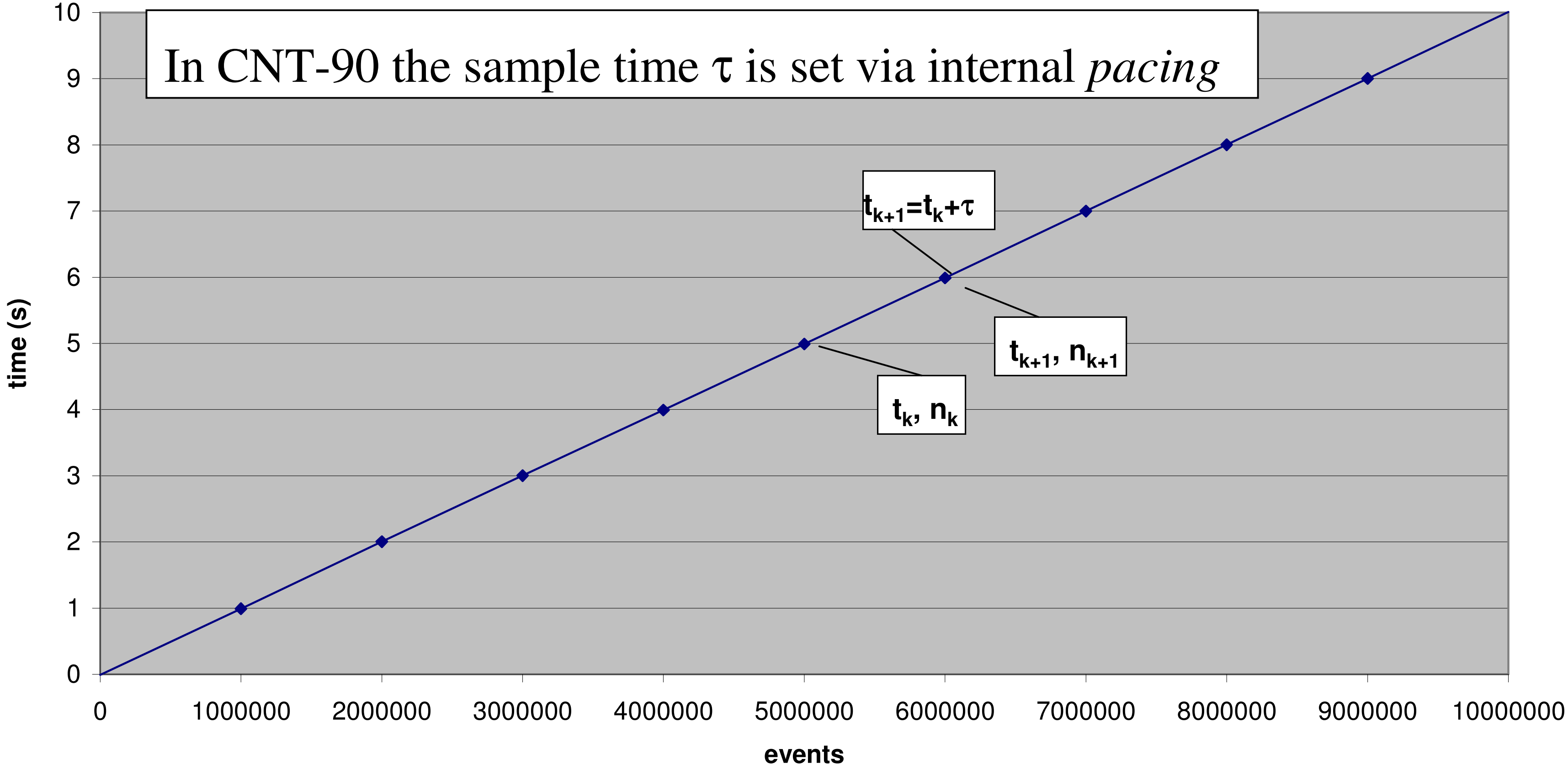
Allan Deviation (τ=1s) back-to-back with timestamping counters



Allan Deviation calculation with timestamping counters

Allan deviation can be calculated from ***pair*** of frequency values or ***triplets*** of time stamp values

Fractional time error: $x(t_k) = t_k(actual) - t_k(ref)$

$$\sigma_y^2(\tau) \approx \frac{1}{2(n-1)} \sum_{k=0}^{k=n-1} (y(t_k+\tau) - y(t_k))^2$$

$$= \frac{1}{2\tau^2(n-1)} \sum_{k=0}^{k=n-2} (x(t_k+2\tau) - 2x(t_k+\tau) + x(t_k))^2$$

CNT-90 Timer/Counter/Analyzer
Continuously timestamping counter

- Increased frequency resolution (regression analysis)
- True back-to-back frequency (correct Allan deviation)